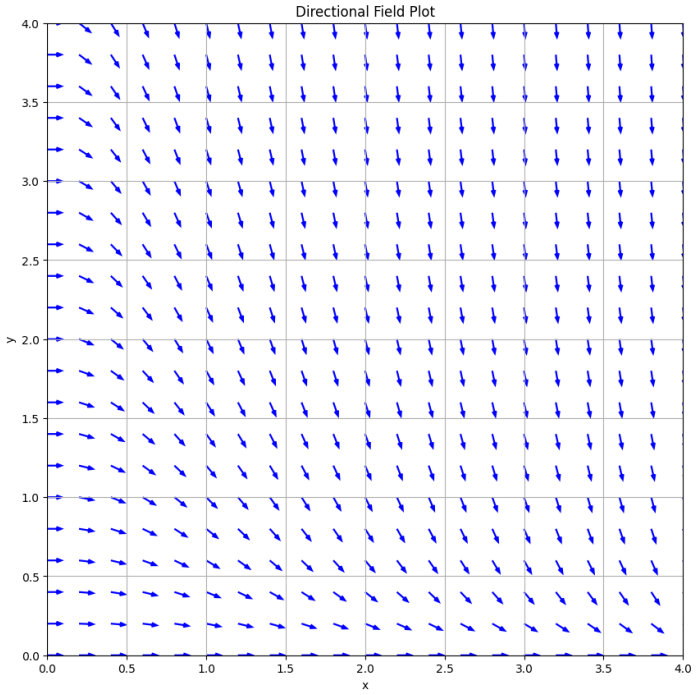
Name: _____ /44

65 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. Consider the DE $y' = -2xy$.

a) *(4 pts)* Its direction field is given. Plot a particular solution satisfying $y(0) = 4$.

b) *(5 pts)* Solve the DE subject to $y(0) = 4$ and find $y(4)$.



2. *(5 pts)* A tank containing 200 litres of water in which 40 g of salt is dissolved. Pure water is then pumped into the tank at a rate of 4 litres per minute; the well-mixed solution is pumped out at the same rate. Find the number of grams of salt $A(t)$ in the tank at time t and the number of grams $A(\infty)$ of salt in the tank as time $t \to \infty$.

3. *(5 pts)* Solve the linear equation $-y' - y/x = 2 - x$ subject to $y(3) = 2$.

4. *(5 pts)* Solve $y'' + 3y = 6 - 2e^{3x}$.

5. *(5 pts)* Find a suitable form of a particular solution $y_p(x)$ of $y^{IV} + 4y'' = 2 + xe^{2x} + \sin 2x$ (do not evaluate constants).

6. *(5 pts)* Verify that the DE $(2x + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0$ is exact and find its general solution.

7. a) *(5 pts)* Find the general solution to $y''' - 4y' = 0$.

b) *(5 pts)* Show that your general solution in part a) is built from a fundamental set of solutions.

Extra credit. *(4 pts)* Draw the phase portrait of the autonomous DE $y' = \cos^2 y + \sin 2y + \sin^2 y$ and classify all critical points.

EXTRA SPACE